Adverse Selection and the Required Return

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An important feature of financial markets is that securities are traded repeatedly by asymmetrically informed investors. We study how current and future adverse selection affect the required return. We find that the bid-ask spread generated by adverse selection is not a cost, on average, for agents who trade, and hence the bid-ask spread does not directly influence the required return. Adverse selection contributes to trading-decision distortions, however, implying allocation costs, which affect the required return. We explicitly derive the effect of adverse selection on required returns, and show how our result differs from models that consider the bid-ask spread to be an exogenous cost.

Understanding the equilibrium expected return — the required return — on securities is the fundamental goal of asset pricing. Of the various factors influencing the required return, this article concentrates on the fact that market participants are asymmetrically informed, and, importantly, that they will also be so at later trading times. What are the costs implied by present and future asymmetric information, and how are they priced? Should a buyer, for instance, offer a reduced price because she would later sell in a market plagued by adverse selection?

In related work, Amihud and Mendelson (1986, 1988) consider the effect of *exogenous* transaction costs. They show that the price of an asset is reduced by the present value of all future trading costs. Said differently, the required return on a security is increased by the per-period transaction cost [see also Constantinides (1986), Vayanos (1998), and Vayanos and Vila (1999)]. In empirical work, the bid-ask spread is often taken as a proxy for this trading cost [see, for instance, Amihud and Mendelson (1986, 1989), Eleswarapu and Reinganum (1993), Chen and Kan (1996), and Eleswarapu (1997)].

We endogenize trading costs as a result of asymmetric information and obtain two main results. First, the required return is elevated by the per-period costs associated with allocation inefficiencies caused by adverse-selection problems.

Second, if agents are symmetric ex ante, then future bid-ask spreads are not a direct trading cost. (Of course, they contribute to the allocation inefficiencies, which are priced as stated by the first result.) That is, their present value does not directly reduce the price — unlike in the case of exogenous trading costs. This result obtains because, in expectation, the future losses an agent will incur when trading due to liquidity reasons are balanced by the gains he will make when trading based on information. If the agents differ ex ante, though, in that some agents are more likely to make liquidity trades than others, then the marginal investor does not break even on average and her expected net trading losses augment the required return.

Our article is related to the literature studying asymmetric information that follows Glosten and Milgrom (1985) and Kyle (1985). While these important articles analyze, among other things, the mechanism determining the bid-ask spread, they cannot address the pricing of future adverse selection. The reason is that these frameworks are characterized by the presence of infinitely liquid agents, who face no risk of a future need to liquidate positions in adverse markets. As a consequence, the required return equals the risk-free return. It is the elimination of these deep-pocketed agents that leads to our results. Our assumption is motivated by the factual observation that in most markets the liquidity of the participants, including market makers,[1] is limited, and agents are subject to such liquidity events as financial distress, hedging or rebalancing needs, tax considerations, agency constraints, etc. Given the possibility of having to liquidate a security position, an agent considers the (allocation) costs associated with the adverse selection prevailing in the market at the future time at which he may need to sell. Moreover, a future buyer also anticipates (allocation) costs when she needs to sell, and so on for the life of the asset.

The model works as follows. A finite number of risk-neutral agents may each own one or zero of a finite set of shares of an asset. Each period, every owner receives a dividend. Then, a randomly chosen agent receives two private signals: one about the next dividend, and one conveying whether he has a (private) cost or benefit of holding the asset (a "liquidity shock"). After these signals are received, all agents can submit market or limit orders, and trade occurs at a market-clearing price. In equilibrium, uninformed owners submit limit orders to sell at the same price, which we

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[1] Indeed, many market makers prefer limited positions overnight, and, in some markets, serve merely as "matchmakers."

call the "ask price," uninformed non-owners submit limit orders to buy, at a price that we call the "bid price," and an informed agent submits a market order or no order.

Limit orders are subject to adverse selection. As a result, the bid price is affected by the well-known "lemons" discount: A market order to sell is a bad signal to other market participants because they know that it may be due to adverse information, which leads to a discounted bid price. Similarly a buyer-initiated trade is good news to the market, and hence it is associated with an analogous premium for the ask price. The sum of the discount and premium constitutes the bid-ask spread.

Our key result, with agent symmetry, is that the adverse-selection discount and premium result is *zero* expected net costs to trading agents. Hence, if the bid-ask spread is generated by adverse selection, then the required return is not *directly* affected by bid-ask spreads. (Adverse selection has an indirect allocation effect, as described below.)

The intuition for this result is as follows. An agent who is selling for liquidity reasons is paid too little relative to his information about the asset value, whereas an agent selling for information reasons is paid too much, as shown by Bagehot (1971), Glosten and Milgrom (1985), and Kyle (1985). In equilibrium, these effects offset each other. Hence, if an agent may trade for either liquidity reasons or information reasons in the future — as is the case in our model — he *anticipates* a future net trading loss of zero. This article innovates by considering the price impact of *future* asymmetric information rather than the extensively studied phenomenon that *current* asymmetric information gives rise to a bid-ask spread.[2]

While not a direct trading cost, adverse selection leads to an *allocation cost* associated with inefficient decisions to buy or sell. This allocation cost is incurred, for instance, when an owner needs cash, but has such good news about the asset value that he (rationally) chooses not to sell. We show that the price of an asset is reduced by the present value of all future net allocation costs, or equivalently, that the required return is increased by the per-period relative allocation cost.

To summarize, adverse selection increases the required return through its allocation costs, not directly through the bid-ask spread. This is consistent with the findings of Easley, Hvidkjær and O'Hara (2002) that cross-sectional returns are explained by the probability of informed trading, but not by the bid-ask spread.

To study the robustness and limitations of our results, we consider several extensions of the basic model: (i) a model with investors that differ

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[2] In a similar spirit, note that the seminal article of Akerlof (1970) studies the way adverse selection affects the market for used cars, leaving open the question of how the prices of new cars are affected by the (anticipated) adverse-selection problems arising after the car is used.

in their likelihoods of becoming informed or of having liquidity needs, (ii) a model with both fixed costs and informational costs, and (iii) a model with risk-averse agents. In the latter two extensions, our main result holds: Adverse selection affects the required return through the allocation costs.

In the first extension, with multiple types of investors, the required return is additionally affected by the marginal investors' expected trading losses. These trading losses occur because a marginal investor is less likely to trade on information than are the best-informed investors. Unless the marginal investor trades only for liquidity reasons, though, these losses are smaller than the bid-ask spread.

This article contributes to the literature on allocation inefficiencies related to adverse selection.[3] In particular, Ausubel (1990) and Eisfeldt (1999) show that real investments in capital are affected by adverse selection, and Diamond and Verrecchia (1991), Wang (1993), and Easley and O'Hara (2000) find that asymmetric information leads to imperfect risk sharing, resulting in reduced prices [see also Akerlof (1970) and Hendel and Lizzeri (1999)]. We find a similar result. However, our simple setting allows us to determine explicitly how future allocation inefficiencies affect prices.

Finally, by isolating the components of future trading costs associated with asymmetric information and characterizing the present value of these costs, and comparing them across different settings, we complement the body of literature — including Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981), Admati (1985), Wang (1993, 1994) — that studies asymmetric information in competitive markets in the rational expectations equilibrium setting.

The rest of the article is organized as follows. Section 1 lays out the basic model. Section 2 presents our results concerning adverse selection. Section 3 develops our extensions. Section 4 concludes.

1. Model

In this section we construct a simple model in which a finite set, $\mathcal{N} = \{1, \ldots, N\}$, of identical risk-neutral agents trade in each of periods $0, 1, \ldots, T - 1 \leq \infty$. We assume that there are K shares of the same asset, with $K > 2$ and $N - K > 2$. Each agent may own one share or no share. Hence, we abstract from the investors' quantity decisions [as do others, such as Glosten and Milgrom (1985)]. While this is a major restriction, it will be clear that the intuition applies more generally.

The timing of events in any period, t, is shown in Figure 1. First, all current owners of an asset receive a dividend. Then, one randomly chosen

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[3] Allocation inefficiency due to adverse selection arises in several corporate finance articles, such as Leland and Pyle (1977), Myers and Majluf (1984), DeMarzo and Duffie (1999), and DeMarzo (2000).
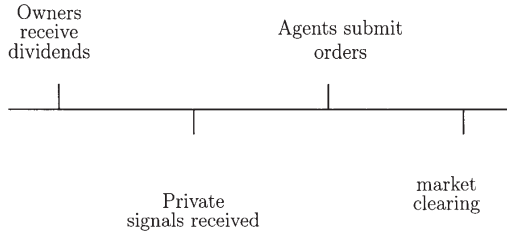
Figure 1
Time-line for period t.

agent, $I(t) \in \mathcal{N}$, receives private information, x_{t+1} and σ_{t+1}. The signals x_{t+1} and σ_{t+1} are independent random variables defined on a given probability space $(\Omega, \mathcal{F}, \mathrm{Pr})$, such that (x_t, σ_t) is i.i.d. over time.[4] The distribution of (x_{t+1}, σ_{t+1}) is common knowledge. The signal x_{t+1} has a strictly positive density on $(\underline{x}, \bar{x})$ (where $\underline{x} < \bar{x}$ are real numbers) and provides information about the dividend next period, $t + 1$. Specifically, we assume that agent $I(t)$ alone knows that the conditional expected value of the dividend next period is x_{t+1}. Our risk-neutrality assumption implies that the conditional utility of an uninformed agent holding the asset is also x_{t+1}. The signal σ_{t+1} is assumed to have a log-concave density and conveys information about agent $I(t)$'s liquidity needs in this period. We model agent $I(t)$'s liquidity needs by assuming that his utility from holding the asset this period is

$$x_{t+1} + \sigma_{t+1}.$$

We note that agent $I(t)$'s utility from holding the asset this period is lower (higher) than the utility derived by the other agents if $\sigma_{t+1} < 0$ ($\sigma_{t+1} > 0$). There are several ways of interpreting the shocks and, more generally, the agents' preferences. For instance, we may assume that there are two goods in the economy: the dividend, which is nonstorable and about which agents may receive preference shocks, and the numeraire, which can be invested at the same rate as the agents' subjective discount rate. Another interpretation, more appropriate for financial assets, is that there is a carrying cost (or benefit) to holding the asset, a cost which may suffer temporary shocks. Such costs could be, among others, financing costs, tax implications, hedging benefits (in a version of the model with risk-averse agents), or could derive from a fund manager's need to rebalance because of in- or outflows of capital. Thus a negative σ_{t+1} could

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[4] We can extend the model to allow the distribution of (x_t, σ_t), that is, the market characteristics, to change (randomly) over time. In each period there would be a public signal about the current state of the economy. See Gârleanu and Pedersen (1999) for a general model with this feature.

represent a need for cash, a high financing cost, or a reduced need for the asset, while a positive σ_{t+1} could represent a state of excess cash, a low financing cost, or an extraordinary need for the asset. We focus on this financial market interpretation, except in Section 3.3, where the two-good interpretation is more natural.

We note that it is not important for the results of the article that a dividend signal and a potential liquidity shock be received during the same period, or by the same agent. What matters is that a trade may originate either from liquidity needs or information.

After agent $I(t)$ has received his private information, there may be trade. The trading mechanism is designed to resemble, stylistically, the opening procedure at the New York Stock Exchange (NYSE).[5] Every agent can submit a limit or market order to buy or sell one share. A limit order specifies a price at which the agent is willing to buy or sell one share (this period), and a market order is interpreted as a limit order with a price of plus or minus infinity. Orders are executed as follows. First, the "specialist" determines the set of prices at which supply equals demand, or at which any excess supply or demand is due to orders at this price. It is easy to see that this set is an interval. The midpoint of this interval is called the clearing price.[6] Then, all trades are executed at the clearing price. If there is an excess supply or demand at the clearing price, then a randomization scheme determines which orders are executed.[7]

Having described the economy and agents' possible actions, we now define formally an equilibrium in this trading game. A strategy for agent i is defined as a process $A = (A_t)_{t=0}^{T-1}$, where $A_t : \Omega \to \{\text{buy}\} \times (-\infty, \infty] \cup \{\text{sell}\} \times [-\infty, \infty) \cup \{\text{notrade}\}$ is measurable with respect to the information,[8] $\mathcal{F}_t^i$, available to player i at time t. A strategy to play, for instance, the action (buy, 17), means that the agent submits a limit order to buy one share for at most \$17. A strategy, A, for agent i is said to be *feasible* if $A_t \in \{\text{sell}\} \times [-\infty, \infty)$ only if agent i is an owner of an asset at the beginning of period t, and if $A_t \in \{\text{buy}\} \times (-\infty, \infty]$ only if agent i is a nonowner.

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[5] Since only one agent is privately informed each period, the trading mechanism is not crucial for our results. One important difference between our model and the NYSE, however, is that the specialist in our model does not take a position.

[6] This interval is always bounded in equilibrium, but to fully describe the game we must also specify the clearing price in the case of an unbounded interval of possible clearing prices: If this interval is bounded below (above), we let the clearing price be the lowest (highest) element of the interval. If the interval is not bounded above nor below (i.e., if there are no limit orders at all), then the clearing price is set at some prespecified value.

[7] This can be interpreted as follows. Orders arrive in a random order, and priority is given based on the time of arrival.

[8] Here, $\mathcal{F}_t^i$ is the σ-algebra generated by

$$(x_1, \sigma_1, \mathit{auction}_0, \ldots, x_t, \sigma_t, \mathit{auction}_{t-1}, x_{t+1} 1_{(I(t)=i)}, \sigma_{t+1} 1_{(I(t)=i)}),$$

where $\mathit{auction}_s$ includes the limit orders at time s and the outcome of that auction.

At time t, agent i chooses a strategy that maximizes the present value of his future cash flows, given that agent j plays strategy A^j for all $j \in \mathcal{N}$, that is, agent i maximizes

$$\Pi_t^i(A^1, \ldots, A^N)$$
$$= E\left[\sum_{s=t+1}^{T} \delta^{s-t}(x_s + \sigma_s 1_{(i=I(s-1))}) 1_{(i \in O_s)} - \sum_{s=t}^{T-1} \delta^{s-t} z_s^i \,\middle|\, \mathcal{F}_t^i \right], \qquad (1)$$

where z_s^i is the net cash payment[9] (due to sales or purchases of the asset) made by agent i at time s, $O_s \subset \mathcal{N}$ is the set of owners, and $\delta > 0$ is the agents' subjective time-discount factor.

Definition 1. *An equilibrium is a set of feasible strategies, $A = (A^1, \ldots, A^N)$, for the respective agents in $\mathcal{N}$, such that, for all $i \in \mathcal{N}$,*

$$E(\Pi_0^i(A)) \geq E(\Pi_0^i(A^1, \ldots, A^{i-1}, A', A^{i+1}, \ldots, A^N)), \qquad (2)$$

for all strategies A' feasible for agent i.

Since we are interested in adverse selection, not in reputation effects, we consider only (symmetric) Markov equilibria, that is, equilibria in which any agent's strategy at time t is a function of whether or not he is an owner, and, if he is informed, of (x_{t+1}, σ_{t+1}). Consequently the agents' optimal strategies can be characterized using dynamic programming, with the ownership status as the only state variable. To do this we define continuation value functions. The continuation value at time t, after the dividend is paid and before information is received, is denoted by S_t for the owners and by B_t for the nonowners. That is,

$$S_t = E(\Pi_t^i(A) \,|\, i \text{ owner at time } t) \qquad (3)$$

$$B_t = E(\Pi_t^i(A) \,|\, i \text{ nonowner at time } t). \qquad (4)$$

2. Asymmetric Information and the Required Return

In this section we solve for the equilibrium and determine how asset prices, or equivalently, required returns, are affected by the adverse-selection problem.

We analyze the symmetric equilibrium in which all uninformed owners submit the same limit order to sell, which we call the "ask price," and all uninformed nonowners submit the same limit order to buy, which we call

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[9] More precisely, z_s^i is defined as follows. If agent i sells (buys) at time s with a clearing price of P_s, then $z_s^i = P_s(z_s^i = -P_s)$, and $z_s^i = 0$ if agent i does not trade.

the "bid price".[10] The informed agent submits a market order or refrains from trading.

To determine the exact strategies, we consider first the bid price that uninformed nonowners offer, taking as given the behavior of the informed agent and of the owners. An uninformed nonowner does not submit a market order — as we later show formally — because of the adverse-selection problem. Instead, each uninformed nonowner submits a limit order to buy one share at his reservation value (as in Bertrand competition). The reservation value is the expected next dividend conditional on a sale by the informed owner, plus the value of being an owner next period, reduced by the (opportunity cost associated with the) value of being a nonowner next period. Hence the bid price is

$$\text{bid}_t = \delta(\hat{x} + S_{t+1} - B_{t+1}), \tag{5}$$

where $\hat{x} = E(x_{t+1} \mid \text{sale})$ is the expected next dividend given that the informed owner submits a market order to sell. [This informal definition of $\hat{x}$ is made precise in Equation (7).]

Now, consider an informed owner's decision. If he wants to sell, then the best price, he can get is the bid price, since all the nonowners submit limit orders. Therefore his decision comes down to keeping the asset or submitting a market order to sell.[11] He sells if and only if this gives him a higher continuation value than that obtained by keeping the asset, that is, if

$$\text{bid}_t + \delta B_{t+1} \geq \delta(x_{t+1} + \sigma_{t+1} + S_{t+1}),$$

which, using Equation (5), is simplified to

$$x_{t+1} + \sigma_{t+1} \leq \hat{x}. \tag{6}$$

Hence an informed owner sells if his news about the dividend is worse than a cutoff level, which depends on his liquidity need.

Given these strategies, equilibrium is characterized by the condition that the uninformed nonowners' expectations are consistent with an informed owner's actions, that is,

$$\hat{x} = E(x_{t+1} \mid x_{t+1} + \sigma_{t+1} \leq \hat{x}), \tag{7}$$

where the right-hand side is well defined if $\Pr(x_{t+1} + \sigma_{t+1} \leq \hat{x}) > 0$, and otherwise we define it as the lower bound, $\underline{x}$, of the support of x_{t+1}. The

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[10] It can be shown that there are no equilibria in which all uninformed owners or nonowners place market orders. Thus, in all other symmetric equilibria, uninformed nonowners or uninformed owners (or both) place no order at all. The equilibrium that we focus on is thus the most efficient symmetric equilibrium, and essentially the only symmetric equilibrium in which uninformed agents may buy as well as sell.

[11] Of course, he could equivalently submit a low sell-limit order that hits the buy-limit orders. We consider this the same as a market order.

following proposition derives some important properties of $\hat{x}$; the full characterization of the equilibrium is given in Proposition 2 below.

Proposition 1. (*i*) *There exists a solution to Equation* (7). (*ii*) *Any solution is in the interval* $[\underline{x}, \mu]$, *where* $\mu = E(x_t)$. (*iii*) *If* $Pr(\sigma_t < 0) = 0$, *the only solution is* $\underline{x}$; *otherwise all solutions are in* $(\underline{x}, \mu]$. (*iv*) μ *is a solution if and only if* $Pr(\sigma_t \in [-\infty, -(\bar{x} - \mu)]) > 0$ *and* $Pr(\sigma_t \in [-(\bar{x} - \mu), \mu - \underline{x}]) = 0$.

The first part of Proposition 1 shows that there always exists an equilibrium (subject to our later verification that only informed agents place market orders). The second part shows that, in equilibrium, the expected next dividend conditional on a market order to sell is lower than the average dividend because of the adverse-selection problem. The third part shows that, if there is no liquidity reason to sell, the adverse selection leads to a market breakdown, as in Akerlof (1970), but with liquidity motives to sell, trade happens in equilibrium. The fourth part shows that there is a real adverse-selection problem ($\hat{x} < \mu$) unless all liquidity shocks are so large that an informed owner with a negative liquidity shock always sells (regardless of his information about the dividend) and an informed owner with a positive shock never sells. We assume the more realistic situation in which an informed agent need not have an (extreme) liquidity shock, that is, $Pr(\sigma_t \in [-(\bar{x} - \mu), \mu - \underline{x}]) > 0$.

The equilibrium strategies of uninformed owners and an informed nonowner are derived analogously. The uninformed owners submit limit orders to sell at their reservation value,

$$\text{ask}_t = \delta(\check{x} + S_{t+1} - B_{t+1}), \tag{8}$$

where $\check{x} = E(x_{t+1} \mid \text{buy})$ is the expected next dividend given that someone submits a market order to buy. An informed nonowner submits a market order to buy if $x_{t+1} + \sigma_{t+1} \geq \check{x}$, and otherwise he does not submit an order. The equilibrium condition for $\check{x}$ is

$$\check{x} = E(x_{t+1} \mid x_{t+1} + \sigma_{t+1} \geq \check{x}), \tag{9}$$

and existence of equilibrium follows from a simple application of Proposition 1.

Finally, we verify that the uninformed traders optimally submit limit orders as specified above. It suffices to consider an uninformed owner. An owner must do one of the following: (i) submit a market order to sell, (ii) submit a limit order to sell, or (iii) submit no order.

If an uninformed owner submits a market order, he is sure to sell his share. If there is no other market order, he is selling at the bid price. If there is a buy market order, then the owner is selling at the midprice. Intuitively, this is an unprofitable strategy, since the owner is selling at a

low price that reflects the buyers' fear of information-based sales, while the owner has no adverse information. We provide a formal argument in the appendix.

If the owner submits a limit order (higher than the bid), then he sells only if his order is hit by a market order. If the owner's order is hit, the buyer must be an informed agent, and therefore the owner's reservation value is, as explained above, $\text{ask}_t = \delta(\check{x} + S_{t+1} - B_{t+1})$. Hence the owner's equilibrium action — a sell-limit order at the ask price — is indeed optimal. (The owner is indifferent between this action, a greater limit order, or no order at all.)

Proposition 2 summarizes the structure of equilibria with trade, and characterizes the value functions.

Proposition 2. *Suppose $\hat{x}$ and $\check{x}$ solve Equations (7) and (9), respectively. Then the following strategies constitute an equilibrium. Uninformed non-owners and owners submit limit orders at the bid [Equation (5)] and ask [Equation (8)], respectively. An informed owner submits a market order to sell if $x_{t+1} + \sigma_{t+1} \le \hat{x}$, and an informed nonowner submits a market order to buy if $x_{t+1} + \sigma_{t+1} \ge \check{x}$. The value functions are given by*

$$S_t = \sum_{s=t+1}^{T} \delta^{s-t}(\mu + \sigma^+ - \hat{c}) \tag{10}$$

$$B_t = \sum_{s=t+1}^{T} \delta^{s-t}(\sigma^+ - \check{c}), \tag{11}$$

where

$$\sigma^+ = \frac{1}{N} E(\sigma_t 1_{(\sigma_t > 0)}) \tag{12}$$

$$\hat{c} = \frac{1}{N} E\left(\sigma_t 1_{(\sigma_t > 0, x_t + \sigma_t \le \hat{x})} - \sigma_t 1_{(\sigma_t < 0, x_t + \sigma_t > \hat{x})}\right) \tag{13}$$

$$\check{c} = \frac{1}{N} E\left(\sigma_t 1_{(\sigma_t > 0, x_t + \sigma_t < \check{x})} - \sigma_t 1_{(\sigma_t < 0, x_t + \sigma_t \ge \check{x})}\right). \tag{14}$$

The value of ownership is the expected present value of future dividends, $\mu = E(x_t)$, and efficient private benefits, σ^+, reduced by the present value of the future "allocation cost," $\hat{c}$. The efficient private benefits stem from the possibility of owning the asset when one has a special need for it, that is, when $\sigma > 0$. The allocation cost is due to the possible misallocation of the asset — namely, it may be held by an agent with a negative private value, or may be sold by an agent with a positive private value. The former

happens when the owner has a need for cash and at the same time such good news about the next dividend that he chooses not to sell. The latter happens when an owner who has a special need for the asset simultaneously has bad news about the next dividend. The value of being a nonowner comes from the expected efficient private benefits, σ^+ — buying when one has a special need for the asset — reduced by the (buy-side) allocation cost, $\check{c}$. The buy-side allocation cost reflects the possibility that either of the following two situations occurs: a nonowner has positive private value, but not a good enough signal to induce purchase; a nonowner has negative private value, but such a good signal that he buys anyway. In all the terms above, the factor $1/N$ represents the probability that a given owner, respectively nonowner, will have a liquidity shock. These probabilities enter the price formula in the same way in a more general model in which several agents may be subject to shocks. In particular, the effects of the allocation costs need not decrease as the market size (N) increases, provided that the fraction of agents with liquidity shocks stays constant.

Knowing the value functions, we can compute the equilibrium prices explicitly as

$$\text{bid}_t = -\delta(\mu - \hat{x}) + \sum_{s=t+1}^{T} \delta^{s-t}\mu - \sum_{s=t+2}^{T} \delta^{s-t}(\hat{c} - \check{c}) \tag{15}$$

$$\text{ask}_t = \delta(\check{x} - \mu) + \sum_{s=t+1}^{T} \delta^{s-t}\mu - \sum_{s=t+2}^{T} \delta^{s-t}(\hat{c} - \check{c}), \tag{16}$$

where $\check{x} \leq \mu \leq \hat{x}$. We compare the bid and ask prices to their counterparts in a full-information economy. With full information, the bid and ask prices are the same, and their expected value is

$$E\left(\delta x_{t+1} + \sum_{s=t+2}^{T} \delta^{s-t}\mu\right) = \sum_{s=t+1}^{T} \delta^{s-t}\mu. \tag{17}$$

We note that private values are not priced in a competitive full-information market since at most one agent has this private benefit and there are several assets.[12]

The bid price differs from the average full-information price for two reasons. First, there is a "lemons discount," $\mu - \hat{x} > 0$, because a market order to sell may be due to bad news about the next dividend. Second, the bid price is affected by the inefficient allocations that arise because of the adverse-selection problem. It is noteworthy that the price is reduced by

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[12] The full-information value functions are as in Equations (10) and (11) with $\hat{c} = \check{c} = 0$.

the sell-side allocation cost — an owner pays less in anticipation of incurring these costs — but is increased by the buy-side allocation cost — an owner is happy to pay more in order to avoid the buy-side allocation costs. The sign of the effect depends on the distribution of (x_t, σ_t).

The bid price contains just a single term related to the lemons discount, and a whole series of allocation costs. To explain this, we consider an agent buying an asset. How should he be compensated for the fact that he may be selling the asset in the future at the bid price? Conditional on selling because of a need for cash ($\sigma < 0$), the agent will be paid too little because of the lemons discount. Conditional on selling for information reasons ($\sigma \geq 0$), however, he will be paid too much. In equilibrium these effects balance. Therefore future lemons discounts do not affect the current price directly. The future adverse-selection problems do affect the agent, though, through the allocation inefficiencies that they imply.

Similarly the ask price is elevated by a "peach premium," $\check{x} - \mu > 0$, since a market order to buy may be caused by positive news about the asset. Also, the ask price is affected by the allocation costs in the same way as the bid price is.

The bid-ask spread, $\delta(\check{x} - \hat{x}) > 0$, is the difference between the lemons discount and the peach premium, and is positive. In equilibrium, the bid-ask spread does not, on average, impose a cost on the agent submitting a market order: a buyer is paying a peach premium, but indeed is getting a peach; a seller receives a lemons discount, but indeed is selling a lemon. The bid-ask spread does, however, inhibit some efficient trades, and in this sense it is a cost.

The case of symmetry is an interesting benchmark:

Proposition 3. *Suppose x_t and σ_t are symmetrically*[13] *distributed around μ and 0, respectively. Then, there exists an equilibrium with $\hat{c} = \check{c}$ and $\mu - \hat{x} = \check{x} - \mu$. In this case, the mid price is equal to the average full-information price, and has an expected return equal to the risk-free rate, $r = 1/\delta - 1$.*

This proposition implies that if x_t and σ_t are symmetrically distributed around zero, then the mid price (1/2 bid + 1/2 ask) is the same as the average price with no asymmetric information, and at the same time there is a strictly positive bid-ask spread. More generally, Equations (15) and (16) show that whether the full-information price is between the bid and ask prices depends on the expected future allocation costs, $\sum_{s=t+2}^{T} \delta^{s-t}(\hat{c} - \check{c})$.

We can express our results in terms of the required rate of return. Because of the bid-ask spread, returns can be measured in various ways. A particularly simple expression obtains if we consider the following

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[13] We say that a random variable, say x_t, is symmetrically distributed around μ if $x_t - \mu$ and $-(x_t - \mu)$ have the same distribution.

weighted average of the bid and ask prices: $p_t := \frac{\check{x}-\mu}{\check{x}-\hat{x}}\text{bid}_t + \frac{\mu-\hat{x}}{\check{x}-\hat{x}}\text{ask}_t$. The required rate of return, measured in terms of p_t, is

$$\frac{p_{t+1}+\mu}{p_t} - 1 = r + \frac{\delta(\hat{c}-\check{c})}{p_t}, \tag{18}$$

that is, the risk-free return r plus a premium related to the allocation costs.[14]

As discussed in the introduction, these results should be seen in contrast to the literature on exogenous transactions costs [see Amihud and Mendelson (1986,1988), Constantinides (1986), Vayanos (1998), Vayanos and Vila (1999), and others], which finds that the price of an asset should be reduced by the present value of all future trading costs, or equivalently, the required rate of return should be increased by the amortized transactions costs. For stocks on the NYSE, the average bid-ask spread is approximately 2.2%, and the average turnover is approximately 60%.[15] Hence this literature would suggest that the average required rate of return should be increased by about $0.6 \cdot 2.2\% = 1.3\%$ because of trading costs, which would have a large impact on the level of prices. We find, on the other hand, that if bid-ask spreads are generated (partly) by adverse selection, then the price impact would be smaller than that. Hence our result might help explain the rather modest price impact associated with the reduction in bid-ask spreads of about 40% that occurs when a stock changes its exchange listing from NASDAQ to NYSE [see Barclay, Kandel and Marx (1998) and Elyasiani, Hauser and Lauterbach (2000)].

As an example, Figure 2 illustrates the difference between the excess return that obtains with an exogenous bid-ask spread (dashed line) and with the endogenous spread implied by our model (solid line). To make this graph, we vary the dividend uncertainty and, for each level of uncertainty, we compute the bid-ask spread and required excess return in our model. Further, we calculate the excess returns implied by an exogenous bid-ask spread of the same magnitude and with the same trading frequency as implied by our model. We assume that one model period is half a month, but we annualize the excess returns.

3. Extensions

We consider three extensions of the basic model. We study models with multiple types of investors, with both informational costs and fixed costs of trading, and with risk-averse agents.

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[14] Other ways of measuring the return are to some extent affected directly by the bid-ask spread. In particular, the price $p_t := \alpha\text{bid}_t + (1-\alpha)\text{ask}_t$ implies a return of $\frac{p_{t+1}+\mu}{p_t} - 1 = r + \frac{\delta(\hat{c}-\check{c})}{p_t} - \frac{r\delta(\alpha(\check{x}-\hat{x})-(\check{x}-\mu))}{p_t}$. Note that the coefficient r on the bid-ask spread confers it a second-order effect, unlike elsewhere in the literature.

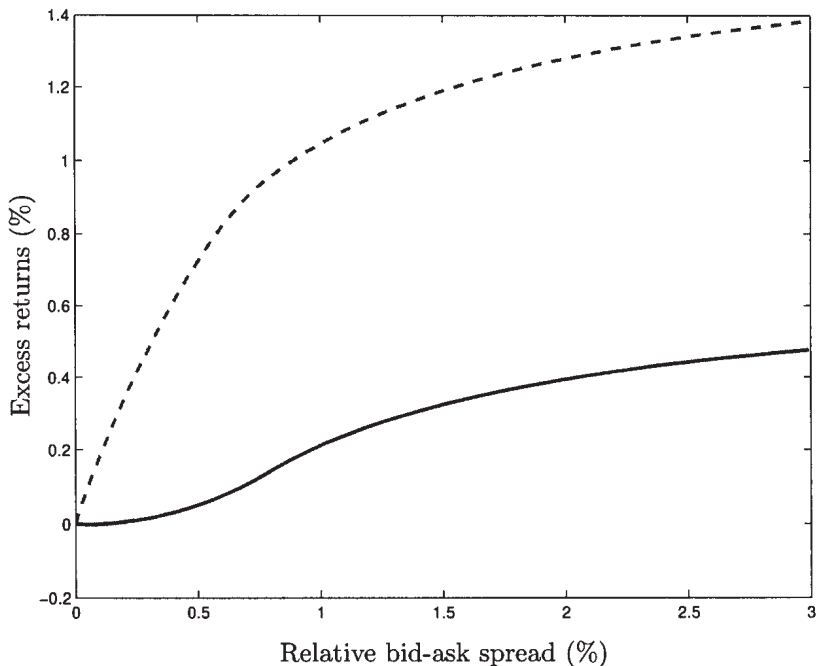[15] These numbers are from Chalmers and Kadlec (1998).

Figure 2
The solid line shows our information-based model's implied excess return for varying levels of dividend uncertainty (v), while the dashed line shows the excess return based on the same trading frequency and bid-ask spread, but treating the spread and trading decision as exogenous. The model period is half a month, the parameters are $T = \infty$, $\delta = \exp(-0.10/24)$, $x \sim u([0.42 - v, 0.42 + v])$, $\sigma \sim u([-0.83, 0.13])$, $N = 13$, $K = 4$, and the returns are annualized by multiplying by 24.

3.1 Multiple investor types

The analysis of Section 2 relied on the assumption of symmetric agents. A key implication of symmetry is that the lemons discount received by a seller is consistent with his chance of selling on bad news. If the seller is less likely to be informed than the average trader, however, then this is no longer true. Hence differences in investors' chances of receiving information or liquidity shocks may have pricing implications. This section shows that the required return is elevated by the marginal investor's expected trading losses to the better informed investors.

To keep the analysis as simple as possible, we assume that every period owners receive a dividend, then agents trade, then an agent receives information, and finally agents may trade again. The difference from Section 1 is the additional round of trading preceding the acquisition of information. This round (which can be allowed in the basic model without any changes, since it would not be used) is introduced here to avoid the complication of time variation in the likelihood that a sale is made by one

type or another. The intuition we want to capture does not concern the variation in these likelihoods.

There are two types of investors, N_1 investors of type 1, and N_2 investors of type 2, and hence a total of $N = N_1 + N_2$ investors. (This can be generalized to more investor types.) In any period, t, one randomly chosen investor receives private information, $x_{t+1} \in [\underline{x}, \bar{x}]$, about the dividend next period. If the chosen agent is of type 2, then he also receives a private signal, σ_{t+1}, reflecting his liquidity need, as in the basic model, that is, his utility for holding this period is $x_{t+1} + \sigma_{t+1}$. All liquidity shocks are negative: $\sigma_{t+1} \leq 0$. (In effect, this convenient simplifying assumption restricts attention to the adverse-selection problem faced by sellers, while capturing the intuition.) Type 1 investors never have liquidity shocks, and therefore they have a comparative advantage in holding the asset.[16] Hence, if there are more type 1 investors than assets, then only those investors participate in the market, and we are back to the model of Section 2. Here we assume that there are fewer type 1 agents than assets, or, more precisely, that $0 < N_1 < K-1$.

The following strategies constitute an equilibrium. In the first round of trading of any period t, owners of type i submit orders to sell at their reservation value, $S_t^i - B_t^i$, where S_t^i and B_t^i are the reservation values of owners and nonowners, respectively, of type i. Type 2 nonowners submit limit orders to buy for $S_t^2 - B_t^2$, and type 1 nonowners submit market orders to buy. The bid, ask, and clearing prices all equal $S_t^2 - B_t^2$, and after this round of trade, all type 1 agents own an asset (which is the purpose of introducing the first round of trading).

In the second round of trading, agents have asymmetric information. Uninformed type 2 nonowners submit limit orders to buy a share at their reservation value,

$$\text{bid}_t = \delta(\hat{x} + S_{t+1}^2 - B_{t+1}^2), \tag{19}$$

where $\hat{x} = E(x_{t+1} \mid \text{sale})$.

An owner with bad news or a liquidity need sells his asset. More precisely, an informed type 1 agent sells if $x_{t+1} \leq \hat{x}$, and an informed type 2 agent sells if $x_{t+1} + \sigma_{t+1} \leq \hat{x}$. Hence the equilibrium condition is

$$\hat{x} = \gamma E(x_t \mid x_t \leq \hat{x}) + (1 - \gamma)E(x_t \mid x_t + \sigma_t \leq \hat{x}),$$

where

$$\gamma = \text{Pr}(\text{sale by type 1} \mid \text{sale})$$

$$= \frac{N_1 \text{Pr}(x_t \leq \hat{x})}{N_1 Pr(x_t \leq \hat{x}) + (K - N_1)\text{Pr}(x_t + \sigma_t \leq \hat{x})}.$$

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[16] It is straightforward to allow agents to also differ in their likelihood of being informed.

Naturally the lemons discount is the weighted average of the adverse selection associated with sales by agents of type 1 and 2, respectively. The weights reflect the relative likelihood that a sale is initiated by each type of agents.

The assumption $\sigma_t \leq 0$ means that there are no liquidity reasons to buy, and therefore agents assume that possible buy orders are motivated by private information. Hence an uninformed owner of type i submits a limit order to sell at her reservation value, $\delta(\bar{x} + S_{t+1}^i - B_{t+1}^i)$, given the best possible signal, $\bar{x}$, about the next dividend.

These considerations show the following.

Proposition 4. *In the equilibrium described above, the bid and ask prices are*

$$\text{bid}_t = -\delta(\mu - \hat{x}) + \sum_{s=t+1}^{T} \delta^{s-t}\mu - \sum_{s=t+2}^{T} \delta^{s-t}(\hat{c} + \theta[\hat{\hat{x}} - \hat{x}])$$

$$\text{ask}_t = \delta(\bar{x} - \mu) + \sum_{s=t+1}^{T} \delta^{s-t}\mu - \sum_{s=t+2}^{T} \delta^{s-t}(\hat{c} + \theta[\hat{\hat{x}} - \hat{x}]),$$

where $\hat{c} = -E(\sigma_t 1_{(x_t + \sigma_t > \hat{x})})/N$ is the allocation cost for type 2 agents, $\theta = Pr(x_t + \sigma_t \leq \hat{x})/N$ is their probability of submitting a market order to sell, and $\hat{\hat{x}} = E(x_t \mid x_t + \sigma_t \leq \hat{x})$.

The prices are as in the basic model, except for the last term involving $\hat{\hat{x}} - \hat{x} > 0$. This last term is due to the fact that the "marginal" (type 2) investor on average is paid too little when he is selling. Conditional on a type 2 investor selling, the expected dividend is $\hat{\hat{x}}$. Buyers do not know, however, if a sale is initiated by a type 2 investor. Buyers know that a sale could also be initiated by a type 1 agent, in which case the expected dividend is $E(x_t \mid x_t \leq \hat{x})$. Hence, the lemons discount, $\hat{x}$ (in the bid price), reflects the average adverse selection over sales by type 1 and 2 agents. Therefore the part $\hat{\hat{x}} - \hat{x} = \gamma(E[x \mid x + \sigma < \hat{x}] - E[x \mid x < \hat{x}])$ of the lemons costs, which is due to the adverse selection related to the inframarginal owners, is a cost to the marginal investor. The prices are reduced by the present value of all these future trading costs, or equivalently, the required rate of return is increased by $\delta\theta(\hat{\hat{x}} - \hat{x})/p_t$.

It is instructive to consider the two polar cases. On the one hand, if type 1 investors suffer from the same liquidity shocks as type 2 investors, then $\hat{\hat{x}} = \hat{x}$ and the lemons discount translates into no trading cost. On the other hand, if the marginal investor is never informed, then the entire lemons discount, $\mu - \hat{x}$, is a trading cost to him, and consequently all future lemons discounts are priced, that is, $\hat{\hat{x}} - \hat{x} = \mu - \hat{x}$.

Glosten and Milgrom (1985) also consider (in their Section 4), the required return if some agents are never informed, while other agents are informed. Glosten and Milgrom (1985) show that an uninformed

investor requires an additional return equal to his expected losses to the informed traders. In their analysis it is puzzling, however, that the market maker, who does not lose to the informed investor, requires the same return as the uninformed, and further puzzling that the informed trader requires this return.

We strengthen the result in three ways: (i) by deriving it in a model in which all agents solve well-defined dynamic problems, (ii) by showing how it depends on the (ex ante) differences among agents, and (iii) by showing how required returns are, additionally, affected by allocation costs.

3.2 Fixed costs

In this section we study how the presence of fixed trading costs affect adverse selection. These costs can be due to order processing costs, search costs, or rents extracted by the market maker.

Suppose that agents must pay a fixed transaction cost, c, when they buy or sell. Then the bid and ask prices are

$$\text{bid}_t = \delta(\hat{x} - c + S_{t+1} - B_{t+1}) \tag{20}$$

$$\text{ask}_t = \delta(\check{x} + c + S_{t+1} - B_{t+1}), \tag{21}$$

where $\hat{x}$ and $\check{x}$ are the expected next dividend conditional on a market order to sell or buy, respectively, with equilibrium conditions

$$\hat{x} = E(x_t \mid x_t + \sigma_t \le \hat{x} - 2c)$$
$$\check{x} = E(x_t \mid x_t + \sigma_t \ge \check{x} + 2c).$$

(Proposition 1 gives conditions for the existence of equilibrium by letting $\sigma_t \pm 2c$ play the role of σ_t.) The value functions are now

$$S_t = \sum_{s=t+1}^{T} \delta^{s-t}(\mu + \sigma^+ - \hat{c} - \hat{\theta}2c) \tag{22}$$

$$B_t = \sum_{s=t+1}^{T} \delta^{s-t}(\sigma^+ - \check{c} - \check{\theta}2c), \tag{23}$$

where $\hat{\theta}$ and $\check{\theta}$ are the probabilities of submitting a market order to sell or buy, respectively,

$$\hat{\theta} = \frac{1}{N}\Pr(x_t + \sigma_t \le \hat{x} - 2c)$$
$$\check{\theta} = \frac{1}{N}\Pr(x_t + \sigma_t \ge \check{x} + 2c),$$

and where σ^+ is given in Equation (12), and $\hat{c}$ and $\check{c}$ are defined analogously to Equations (13) and (14).

The value functions, together with Equations (20) and (21) lead to the following conclusions: First, adverse selection is priced as in the basic model, in that it increases the bid-ask spread and changes the level of prices through its allocation effects. Second, fixed trading costs directly increase the bid-ask spread, and also change the level of prices through the present value of all future trading costs.[17]

We note that the equilibrium volume of trade shrinks as c increases, consistent with the findings of Constantinides (1986) and Vayanos (1998). Also, changes in the fixed transaction cost change the severity of the adverse-selection problem. Higher fixed costs change the average quality of the sold asset, that is, change the lemons cost, $\hat{x}$, and lead to greater allocation inefficiencies.

3.3 Risk aversion
In this section we show that our main results apply in the presence of risk aversion in a particular setting. While the setting is special, it illustrates that our results do not hold only in the case of risk neutrality and helps demonstrate what is needed for our results.

We assume that agents are risk averse with respect to the cash flows of the illiquid assets, with common von Neumann-Morgenstern utility function, $u(\cdot)$. Further, we assume that agents are risk neutral with respect to payments associated with trading the asset. The best interpretation of these preferences is an economy with two goods: houses and apples (the numeraire). If one owns a house, then one must consume its random service flows immediately. Hence agents are risk averse with respect to these service flows. There is a perfect market for apples, which can be invested, and since prices are deterministic, the agents face no risk with respect to the terms of trade for consumption of apples. Similarly Grossman and Laroque (1990) study an economy with assets that yield service flows that must be consumed immediately. These preferences are captured by the following utility function [which replaces Equation (1)] for consumption, $x_s + \sigma_s$, of housing, and consumption, z_s^i, of apples:

$$\Pi_t^i(A^1, \ldots, A^N)$$
$$= E\left[\sum_{s=t+1}^{T} \delta^{s-t} u((x_s + \sigma_s 1_{(i=I(s-1))}) 1_{(i \in O_s)}) - \sum_{s=t}^{T-1} \delta^{s-t} z_s^i \,\middle|\, \mathcal{F}_t^i\right].$$

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[17] Future trading costs reduce prices through a series of terms of the form $(\hat{\theta} - \check{\theta})2c$. The reason that future costs, $\check{\theta}2c$, associated with buying an asset tend to elevate prices is as follows: At some future point in time, an agent may need to own the asset, and if he owns it already, he will save the costs of buying it. In the overlapping-generations (OLG) models of Amihud and Mendelson (1986) and Vayanos (1998), agents are born, buy, hold, sell, and then die. This corresponds most closely to our model with $\hat{\theta} = 0$, and in this case our model delivers the same result as the OLG models: The price is reduced by the present value of all future fixed costs incurred.

The equilibrium of the economy with these preferences is derived similarly to the equilibrium in the basic model, so we report only the resulting prices:

$$\text{bid}_t = -\delta(\bar{u}-\hat{u}) + \sum_{s=t+1}^{T} \delta^{s-t}\bar{u} - \sum_{s=t+2}^{T} \delta^{s-t}(\hat{c}-\check{c})$$

$$\text{ask}_t = \delta(\check{u}-\bar{u}) + \sum_{s=t+1}^{T} \delta^{s-t}\bar{u} - \sum_{s=t+2}^{T} \delta^{s-t}(\hat{c}-\check{c}),$$

where

$$\bar{u} = E(u(x_t))$$

$$\hat{u} = E(u(x_t) \mid u(x_t + \sigma_t) \le \hat{u})$$

$$\check{u} = E(u(x_t) \mid u(x_t + \sigma_t) \ge \check{u})$$

$$\hat{c} = \frac{1}{N} E([u(x_t) - u(x_t + \sigma_t)]1_{(\sigma_t < 0, u(x_t + \sigma_t) > \hat{u})}$$
$$+ [u(x_t + \sigma_t) - u(x_t)]1_{(\sigma_t > 0, u(x_t + \sigma_t) \le \hat{u})}]$$

$$\check{c} = \frac{1}{N} E([u(x_t) - u(x_t + \sigma_t)]1_{(\sigma_t < 0, u(x_t + \sigma_t) \ge \check{u})}$$
$$+ [u(x_t + \sigma_t) - u(x_t)]1_{(\sigma_t > 0, u(x_t + \sigma_t) < \check{u})}).$$

These prices are seen to have the same structure as in the basic model with risk-neutral agents. The effect of risk aversion is that the limit orders involve the certainty equivalent of the next dividend, not the expected dividend, since investors want to be compensated for bearing risk. This is not on average a cost for the informed agent, though, for he is as risk averse as the other agents, and therefore happy to pay for the implicit insurance provided by the fact that the price does not depend on his information about the dividend. As in the risk-neutral model, the price levels are reduced by the present value of future allocation costs.

4. Conclusion

This article studies how adverse selection affects the required return. It shows that the required return is affected by the allocation costs associated with adverse selection and by the bid-ask spread only to the extent that the marginal investor is (ex ante) less likely to be informed than better-informed investors.

There are several limitations to our model. First, agents are restricted to own zero or one unit of the asset. While it seems reasonable that agents can take only limited positions, it is restrictive to assume that agents have no quantity decision within their limits. Intuitively our results seem also

to apply when investors can trade different quantities, but it might be valuable to extend the model to encompass this possibility. This extension would be especially promising if combined with a relaxation of the assumption of exogenous financial distress. In that case, one could imagine that a large security position would be associated with some combination of a high risk of liquidity need and severe illiquidity costs.

Further, we consider only one class of illiquid assets. It would be interesting to consider the asset pricing effects of dynamic trading between multiple classes of illiquid assets.

In our setting, the structure of asymmetric information can easily be made endogenous; that is, the quality of privately learned information can depend on ownership status. The information structure can be further endogenized by assuming that agents must pay for information. In such a generalization, it would indeed be the case (in equilibrium) that the amount of information purchased would depend on ownership, and new implications may arise from studying this endogenous adverse-selection problem.

Appendix A

Proof of Proposition 1.

(*i*) Define $f : \mathbb{R} \to [\underline{x}, \bar{x}]$ by

$$f(y) = E(x_t \mid x_t + \sigma_t \le y), \tag{A.1}$$

where $E(x_t \mid x_t + \sigma_t \le y)$ is defined as $\underline{x}$ for y such that $\Pr(x_t + \sigma_t \le y) = 0$. First, we note that f is continuous because the absolute continuity of x_t implies that $\Pr(x_t + \sigma_t = y) = 0$ for all y, and because of dominated convergence. Second, there exists a y such that $y > f(y)$, since $f(y) \to E(x_t) < \infty$ as $y \to \infty$. Third, $f(\underline{x}) \ge \underline{x}$. Hence, existence of a solution to Equation (7) follows from the intermediate value theorem.

(*ii*) This part of the proposition follows from the fact that $f(y) \le E(x_t)$ for all y. To see that, we note that $y \mapsto E(x_t \mid x_t \le y)$ is increasing toward $E(x_t)$. Further,

$$f(y) = \int E(x_t \mid x_t + \sigma \le y) F(d\sigma \mid x_t + \sigma_t \le y), \tag{A.2}$$

where $F(\cdot \mid x_t + \sigma_t \le y)$ is the distribution of σ_t conditional on $x_t + \sigma_t \le y$, and where the integrand is bounded by $E(x_t)$.

(*iii*) First, if $\Pr(\sigma_t < 0) > 0$, then $f(\underline{x}) > \underline{x}$, which implies that a solution to Equation (7) is greater than $\underline{x}$. Second, if $\Pr(\sigma_t \ge 0) = 1$, then for any $y > \underline{x}$,

$$f(y) \le E(x_t \mid x_t \le y) < y,$$

where the first inequality follows from Equation (A.2), and the second inequality is obvious.

(*iv*) We note that $f(\mu) = \mu$ if and only if $E(x_t \mid x_t + \sigma \le \mu) = \mu$ for almost all σ with respect to the measure $F(\cdot \mid x_t + \sigma_t \le \mu)$. (See the discussion in (*ii*).) Further, $E(x_t \mid x_t + \sigma \le y) = \mu$ if and only if $y - \sigma \ge \bar{x}$. Finally, the support of $F(\cdot \mid x_t + \sigma_t \le \mu)$ is always contained in $(-\infty, \mu - \underline{x}]$. Hence $f(\mu) = \mu$ if and only if $Pr(\sigma_t \in (-\infty, -(\bar{x} - \mu)]) > 0$ and $Pr(\sigma_t \in (-(\bar{x} - \mu), \mu - \underline{x}]) = 0$. ∎

Proof of Proposition 2. We first complete the proof that the agents' strategies form an equilibrium by verifying that uninformed traders do not submit market orders. We distinguish three cases.

(a) The informed trader is also an owner, so that the uninformed seller receives $\delta(\hat{x} + S - B)$, the bid price, for an asset with conditional expected value $\delta(\mu + S - B)$; thus the seller makes an expected profit of $\delta(\hat{x} - \mu) < 0$.

(b) The informed trader is not an owner and does not buy, a case in which the uninformed trader makes an expected conditional profit of

$$\delta(\hat{x} - E(x \,|\, x + \sigma < \check{x})) < \delta(\hat{x} - E(x \,|\, x + \sigma < \hat{x})) = 0.$$

The inequality owes to $\check{x} > \hat{x}$ and to the fact that the density of σ being log-concave implies that x and $x + \sigma$ are affiliated.

(c) The informed trader is a buyer, and the uninformed trader makes $\delta((\hat{x} + \check{x})/ 2 - \check{x}) = \delta(\hat{x} - \check{x})/2 < 0$.

Thus, conditional on being in any of the three cases, a market order is associated with an expected loss.

It remains to derive the value functions. Since

$$\begin{aligned} S_t &= E(\delta[x_{t+1} + \sigma_{t+1}1_{(i=I(t))} + S_{t+1}]1_{(i \text{ keeps asset})} \\ &\quad + [P_t + \delta B_{t+1}]1_{(i \text{ sells})}) \\ &= \delta(\mu + \sigma^+ - \hat{c} + S_{t+1}), \end{aligned}$$

and

$$\begin{aligned} B_t &= E([\delta(x_{t+1} + \sigma_{t+1}1_{(i=I(t))} + S_{t+1}) - P_t]1_{(i \text{ buys})} \\ &\quad + \delta B_{t+1}1_{(i \text{ does not buy})}) \\ &= \delta(\sigma^+ - \check{c} + B_{t+1}), \end{aligned}$$

we get Equations (10) and (11) by recursion. ∎

Proof of Proposition 3. By Proposition 1 there exists a number, $\hat{x}$, that solves Equation (7). Define $\check{x}$ such that $\check{x} - \mu = \mu - \hat{x}$. Then,

$$\begin{aligned} \check{x} - \mu &= \mu - E(x_t \,|\, x_t + \sigma_t \leq \hat{x}) \\ &= \mu - E(x_t \,|\, -(x_t - \mu + \sigma_t) \geq \mu - \hat{x}) \\ &= E(-(x_t - \mu) \,|\, -(x_t - \mu + \sigma_t) \geq \check{x} - \mu) \\ &= E(x_t - \mu \,|\, x_t - \mu + \sigma_t \geq \check{x} - \mu) \\ &= E(x_t \,|\, x_t + \sigma_t \geq \check{x}) - \mu. \end{aligned}$$

Further,

$$\begin{aligned} N(\sigma^+ - \hat{c}) &= E(\sigma_t 1_{(x_t - \mu + \sigma_t > \hat{x} - \mu)}) \\ &= -E(-\sigma_t 1_{(-(x_t - \mu + \sigma_t) < \check{x} - \mu)}) \\ &= -E(\sigma_t 1_{(x_t + \sigma_t < \check{x})}) \\ &= E(\sigma_t) - E(\sigma_t 1_{(x_t + \sigma_t < \check{x})}) \\ &= E(\sigma_t 1_{(x_t + \sigma_t \geq \check{x})}) \\ &= N(\sigma^+ - \check{c}). \end{aligned}$$

∎

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